The Characterization of Lucky Edge Coloring in Graphs

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January 26, 2023

Outline

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Abstract

The lucky edge coloring of graph G is a proper edge coloring which is induced by a vertex coloring such that each edge is labeled by the sum of its vertices. The least integer k for which G has a lucky edge coloring in the set $\{1, 2, \ldots, k\}$ is called lucky number, denoted by $\eta(G)$. The lucky numbers were already calculated for a large number of graphs, but not yet for trees. In this paper, we provide the characterization of lucky edge coloring and calculate the lucky number for graphs which can be regarded as complete m-ary trees.

Outline

Basic Concepts and Notations

A graph G is an ordered pair $(V(G), E(G))$, consisting of

- nonempty set $V(G)$ - set of **vertices**
- $E(G)$ - set of unoredered pair of vertices

(an element of E(G) is called **edge** of G)

convenient to write edge uv instead of edge $\{u, v\}$

Basic Concepts and Notations

$\Delta(G)$ - maximum degree of all vertices in graph G

$\Delta(G) = \max \{d(u) \mid u \in V(G)\}$

$N(u)$ - neighborhood of u

$N(u) = \{v \in V(G) \mid uv \in E(G)\}$

Coloring

vertex coloring - mapping from vertex set assign to set of colors T

$$f : V(G) \to T$$

Normally, take $T = \{1, 2, 3, \ldots, k\}$

edge coloring - mapping from edge set assign to set of colors

$$f : E(G) \to \{1, 2, \ldots, k\}$$

Lucky Coloring

$\mathbb{N}$ - set of positive integers

for a vertext coloring $f : V(G) \to \mathbb{N}$,

the **induced edge coloring** f^*

$$f^* : E(G) \to \mathbb{N}$$

defined by $\quad f^*(uv) := f(u) + f(v) \quad$ for any $uv \in E(G)$

is called **lucky coloring** if f^* is proper coloring

Outline

Theorem 3.1

Let f be a vertex coloring of a graph G. The the following statements are equivalent:

(i) f^* is a lucky coloring of G.

(ii) $f(u_1) \neq f(u_2)$ for all $u_1, u_2 \in N(v)$, $u_1 \neq u_2$ and all $v \in V(G)$.

(iii) $|N(v)| = |f(N(v))|$ for all $v \in V(G)$.

Proposition3.2

Let f be a vertex coloring of a graph G. If f^* is a lucky coloring of G then $\eta(G) > |R_f| \geq \Delta(G)$.

Outline

Remark 4.1

The lucky number of $T_{m,1}$ is $m+1$.

Proposition 4.2

Let $T_{m,2}$ be the complete m-ary rooted tree of hight 2. Then $\eta(T_{m,2}) = 2m+1$.

Proposition 4.3

Let $T_{m,3}$ be the complete m-ary rooted tree of hight 3. Then $\eta(T_{m,3}) = 2m+1$

Proposition 4.4

Let $T_{m,h}$ be an m-ary rooted tree with hight $h \geq 4$. Then $T_{m,h}$ is $(2m+2)$-lucky.

Proposition 4.5

Let $T_{m,h}$ be and m-ary rooted tree with hight h. If $T_{m,h}$ contains a complete m-ary tree with hight greater or equal 4, then $\eta(T_{m,h}) = 2m+2$.

Theorem 4.6

Let $h \geq 4$ and let $T_{m,h}$ be a complete m-ary rooted tree. Then $\eta(T_{m,h}) = 2m + 2$.

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